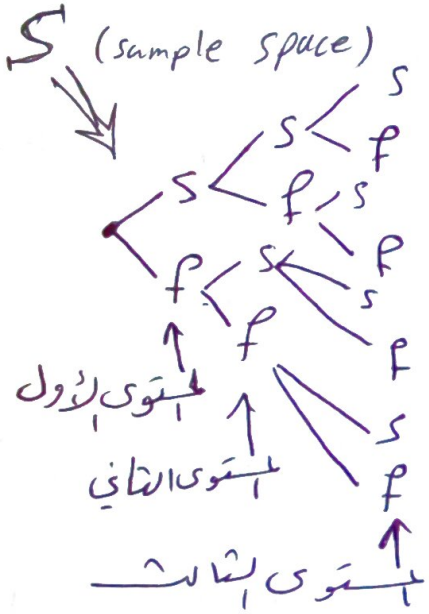
Stochastic

محاضرة [2] د. سيد سلام

الأربعاء 7/10/2015

Tree Diagram



من مثال المحاضرة السابق:

E_1 event all bulbs (burn) Success:
no bulb fails
(SSS)

E_2 // one bulb burns
// two bulbs fail
(SFF), (FSF), (FFS)

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event one bulb fail
E_3 // two bulbs burn
(FSS), (SFS), (SSF)

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event all bulb fail:
E_4 // no bulb succeed
(FFF)

E_5 event first bulb sucess
(SSS), (SSF), (SFF), (SFS)

$E_6 = E_1 \cap E_5$

$E_7 = E_2 \cup E_4$

Turn over

Probabilit of an event :-

- We denote $P(E)$ for the Probability of an event.
- An experiment of selecting a random one of two books A or B. The sample spase $S = [A, B]$

The events are

* No book is selected (impossible event) $E_1 = (\phi_1)$

$$\Rightarrow P(E_1) = 0$$

* book a is selected $E_2(a) \Rightarrow P(E_2) = 1/2$
* book b is selected $E_3(b) \Rightarrow P(E_3) = 1/2$
* book a or b is selected $E_4(a, b) \Rightarrow P(E_4) = 1$

Definition :-

If the simple events of a finite sample Space S are equally probable and $E \subseteq S$.

then $$P(E) = \frac{n(E)}{n(S)}$$

where $n(E) \equiv$ The number of sample points in E

$n(S) \equiv$ " " " " " in S.

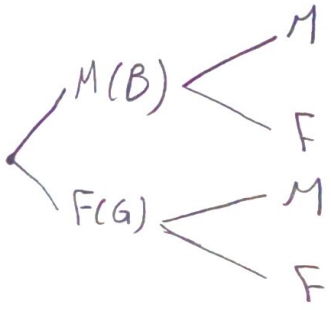
⟹ Turn over

Example: For the experiment "Record the birth of two childs".

- what is the probability that both childs are of the same sex?
- what is the probability that one child at least is a boy?

...>> Solution <<...

$S = (MF), (MM), (FM), (FF)$



$E_1 = \{(MM), (FF)\}$

$E_2 = \{(MF), (MM), (FM)\}$

$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{2}{4} = 1/2$

$P(E_2) = \frac{3}{4}$

$P(E_1 \cap E_2) = P(MM) = 1/4$

$P(E_1 \cup E_2) = P(MM)(MF)(FF)(FM) = 1$

Example: for the experiment "Randomly select and record the members of committe of three from a group of five People" what is the probability of each of the following events:-

E_1: Persons P_1 and P_2 are both selected

E_2: // P_1 & P_2 are selected and P_3 is not selected

E_3: // P_1 is selected and neither P_3 nor P_4 is selected

⟹ Turn Over

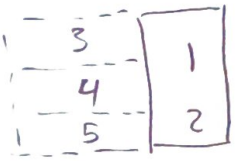
Solution

$S: \{(123), (124), (125), (134), (135), (145), (234), (235), (245), (345)\}$

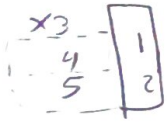
كوافيعه (Combination): هي الحالات بدون تكرار (بدون ترتيب)

$n(S) = 10$ or $n(S) = ({}^5C_3) = \binom{5}{3} = \frac{5*4*3}{3*2*1} = 10$

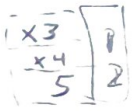
$E_1 = \{(1,2,3), (1,2,4), (1,2,5)\}$



$E_2 = \{(1,2,4), (1,2,5)\}$



$E_3 = \{(1,2,5)\}$



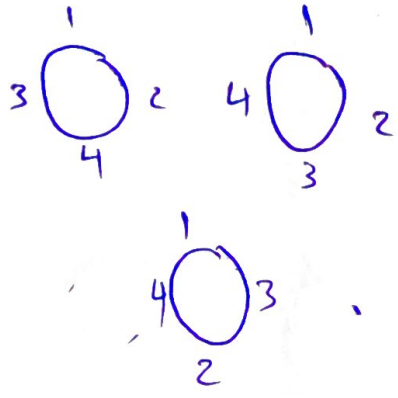
$\therefore P(E_1) = \frac{3}{10}$, $P(E_2) = \frac{2}{10} = \frac{1}{5}$

$P(E_3) = \frac{1}{10}$

Example :- four People are randomly seated at a circular table, find the probability of E_1 and E_2 where:

E_1: P_3 is seated next to P_4

E_2: P_1 is seated next to P_4



$S = \{(1234), (1243), (1324)\}$

$E_1 = \{(1234), (1243)\}$

$E_2 = \{(1234), (1324)\}$

$P(E1) = \quad P(E2) = \frac{2}{3}$

Theorem:-

Let E be an Event in a sample of space S. if E is the empty set, then $P(E) = 0$ if E is not empty, the $P(E)$ = the sum of probabilities of all the simple events which are subsets of E.

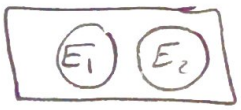
Example: $S = (1, 2, 3, 4, 5, 6)$, $E = (2, 3)$

$$P(E) = P(2) + P(3) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

or $P(E) = \frac{2}{6} = \frac{1}{3}$

Theorem:-

If E_1 and E_2 are events of a sample space S then $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

if E_1 and E_2 are mutually exclusive, $E_1 \cap E = \phi$,

then $P(E_1 \cup E_2) = P(E_1) + P(E_2)$